

An introduction to stochastic solvency capital projections

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Introduction

In this paper, we develop an analytical framework for conducting multi-year solvency projections for insurance companies and explore the added value of incorporating a stochastic modeling approach into this process.

Unlike the traditional deterministic approach, which relies on a single prescribed scenario, the stochastic perspective enables the exploration of a full distribution of possible future outcomes arising from different market and economic risks. This provides a richer understanding of uncertainty and of the interactions between capital markets, liability valuations, and capital requirements.

Adopting a probabilistic view improves visibility over the expected trajectory of the solvency ratio and over the likelihood of breaching regulatory thresholds, reaching dividend-restriction levels, or achieving capital growth.

As such, introducing stochastic modeling offers a more insightful tool for analyzing the risk–return trade-off and for supporting strategic decisions based on quantified probabilities.

The following analysis starts with deterministic Solvency II capital projections for an insurance company and includes the assessment of alternative asset allocation strategies. The analysis is broadened to include stochastic economic scenarios, providing additional insights for the base projections and for assessing alternative asset allocation strategies.

Deterministic projections

The analysis is based on a simplified version of a real local insurance company selling life, health, and non-life business. The investment exposure varies from investment guaranteed products with full investment risk on the company to investment-linked products with minor investment risk on the company.

The Solvency II capital projection model projects forward the economic balance sheet, allowing for real-world economic interest assumptions, sales of new business, dividend distributions, and recalculation of the solvency components for each future projection year.

Key assumptions

ECONOMIC BALANCE SHEET

The main assets and liabilities components of the insurance company, by investment exposure, are shown in Figure 1.

FIGURE 1: ECONOMIC BALANCE SHEET (US BN\$)

		ASSETS	LIABILITIES	TIER 2 CAPITAL	TOTAL CAPITAL
Nostro (Non-Investment Participating)	Life	38.4	20.1		
	Non-Life	38.4	4.8		
	Health	38.4	1.3		
Investment Participating	Life	74.7	73.7		
Total		113.1	100.0	2.0	15.1

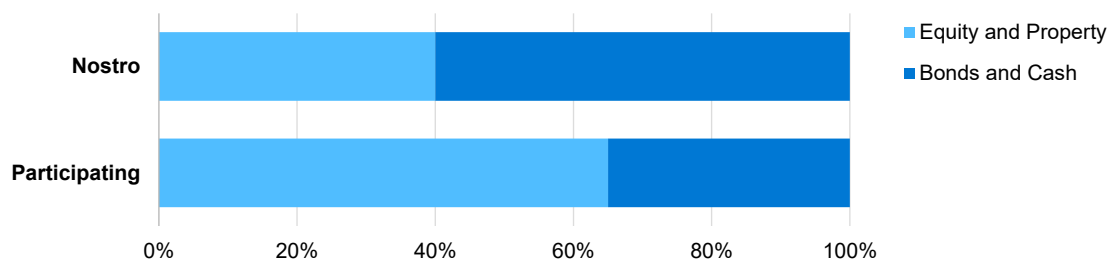
CURRENT SOLVENCY RATIO

The company's current solvency ratio is 150%.

ASSET MIX

The company's current asset position represents a balanced investment strategy, and the split between riskier assets (Equity and Property) and more stable assets (Bonds and Cash) is shown in Figure 2.

FIGURE 2: ASSET ALLOCATION FOR NOSTRO AND INVESTMENT PARTICIPATING PRODUCTS



For the asset allocation above, the expected economic return (real, net of inflation) for each portfolio is shown in Figure 3.

FIGURE 3: EXPECTED ECONOMIC RETURN BY PORTFOLIO FOR THE EXAMPLE IN FIGURE 2

BALANCED	
Nostro	3.5%
Investment Participating	5.1%

In practice, a company's asset allocation has additional granular details, but for the purpose of the analysis, examining the shift between higher-risk and more conservative assets is sufficient.

For simplicity and to highlight the comparison between deterministic and stochastic projections, the following additional assumptions were used:

- **Asset rebalancing.** The asset portfolio is rebalanced annually, such that the asset mix remains constant over time¹
- **Second-order effects.** It is assumed that the economic scenarios do not impact non-economic assumptions, such as lapse rates or sales of new business
- **Complex components.** Components requiring a stochastic calculation themselves, such as the time value of options and guarantees (TVOG), are, for simplicity, assumed to be constant²

Some of the above assumptions may be altered or refined to enhance the modeling; the purpose of the analysis presented here is to demonstrate the benefits of stochastic modeling.

1. Although market movements during the year may temporarily alter the composition of the portfolio, it is assumed that the asset allocation is restored to the target weights as set by the asset allocation strategy.

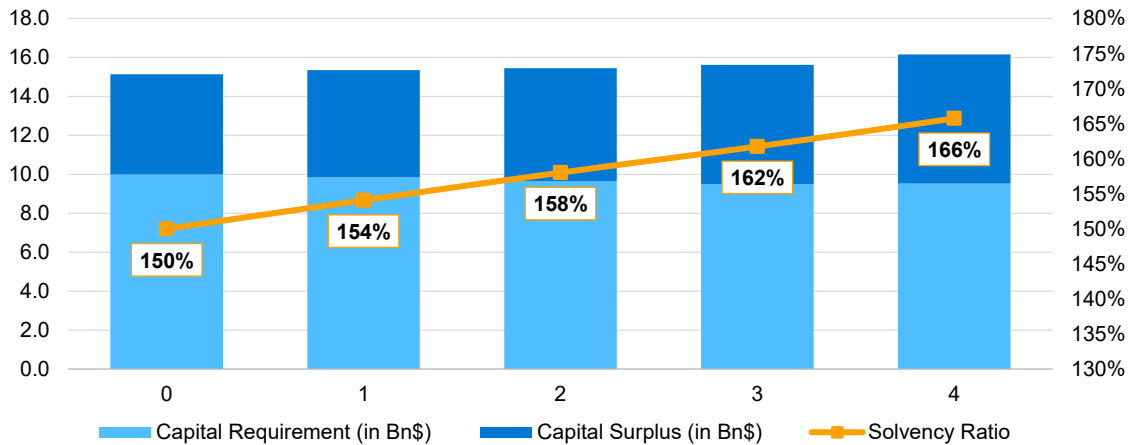
2. A portion of the portfolio contains products with a TVOG, which require stochastic projections to appropriately assess their economic value. To maintain simplicity and focus, we assume a constant, static TVOG value throughout the analysis. This assumption is appropriate given that the portfolio has only limited exposure to such products and based on past experience analysis its value has remained fairly stable.

Base projection

Under the base scenario, the balanced investment strategy is used for the four-year projection horizon.

The results presented in Figure 4 show the projected development of the key solvency components: S2 solvency ratio, Required Capital, and Surplus Capital.

FIGURE 4: SOLVENCY COMPONENTS OVER TIME UNDER A DETERMINISTIC SCENARIO



The capital ratio (after planned dividend distributions) increases gradually from 150% at the start of the projection to 166% at the end of the four-year period. This improvement reflects steady growth in own funds, driven by the release of risk margin, investment earnings above the risk-free rate, and new sales of life, health, and non-life insurance products. A key driver of growth is investment earnings and, as such, should be analyzed in further detail.

Despite the growing portfolio, the solvency required capital requirements remain broadly stable due to the run-off of more capital-intensive products and their replacement with the sale of less capital-intensive products. The stability in capital requirements is also driven by the unchanged balanced investment allocation.

Over the same period, total own funds (capital requirements plus excess capital) increased by approximately USD 1 billion, reaching USD 16 billion after four years.

Assessment of alternative asset allocations

After projecting the solvency ratio with the balanced asset strategy as a benchmark, the company considers two alternative investment strategies—**aggressive** and **conservative** allocation, as shown in Figures 5 and 6.

FIGURE 5: NOSTRO ASSET COMPOSITION

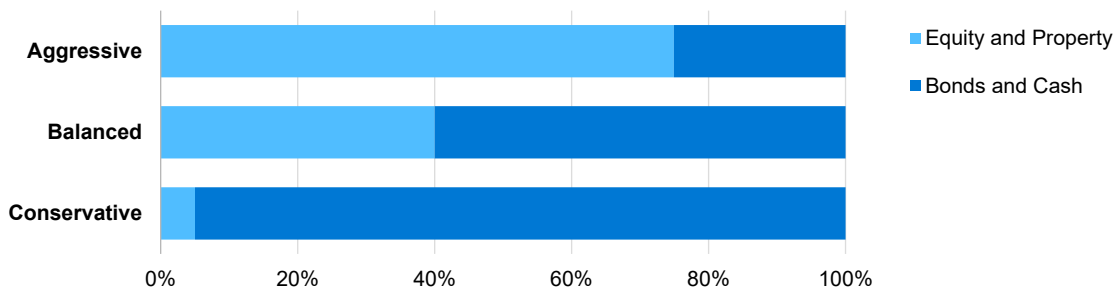
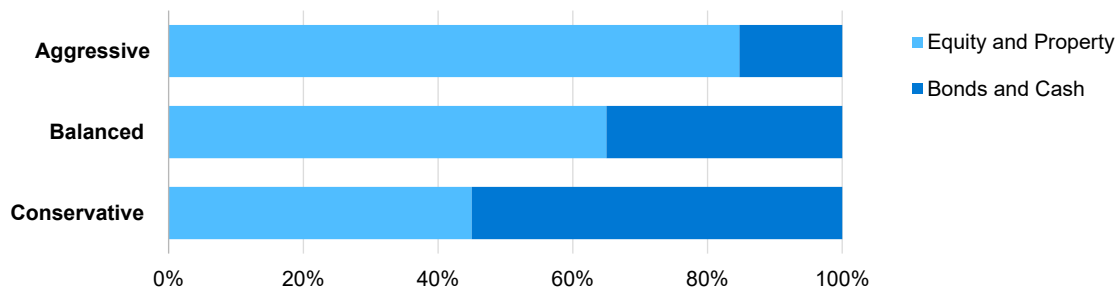


FIGURE 6: PARTICIPATING ASSET COMPOSITION**EXPECTED ECONOMIC RETURNS**

The expected economic return (real, net of inflation) for each portfolio according to the alternative asset allocations is shown in Figure 7.

FIGURE 7: EXPECTED ECONOMIC RETURN BY PORTFOLIO

	BALANCED	AGGRESSIVE	CONSERVATIVE
Nostro	3.5%	5.6%	1.3%
Investment Participating	5.1%	6.2%	4.1%

PROJECTED SOLVENCY II CAPITAL RATIO

In the projection, the different asset allocations broadly affect the results as follows:

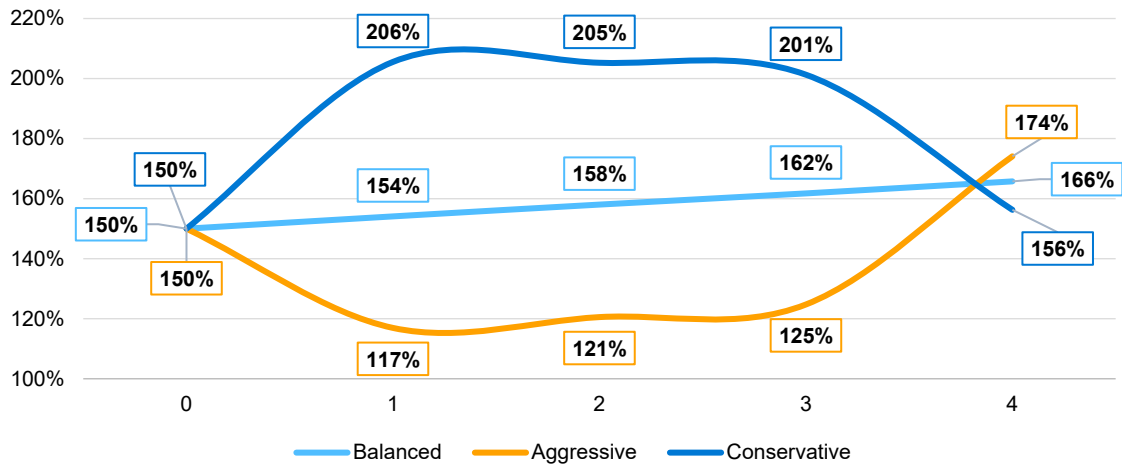
- Investment earnings for the Nostro and Investment Participating fund are calculated using the assumed investment returns and added to the assets
- The Nostro liability cashflows are not affected by the economic returns³
- The investment participating portfolio changes according to the assumed earning rates, and this subsequently affects the demographic capital requirements and risk margin
- The market-risk capital requirements are updated accordingly

To compare these scenarios, in the projection, the company shifts to the new strategy in the first projection year and returns to the balanced asset allocation at the end of four years. Returning to the original asset allocation removes the effect of the different capital requirements and, as such, enables a better comparison using the solvency ratio.

Figure 8 shows the projected solvency ratio over the projection horizon for all three strategies.

3. In this simplistic illustration, the duration of the assets and liabilities have not been matched. For the purposes of this analysis to show the additional value of the stochastic framework, it has been ignored. In further analysis matching of the durations can be explored to the reduce volatility and increase hedging.

FIGURE 8: PROJECTION OF THE SOLVENCY II CAPITAL RATIO



BEHAVIOR OF CAPITAL SURPLUS AND CAPITAL REQUIREMENT

As noted earlier, the aggressive strategy increases market risk and, therefore, raises capital requirements, resulting in an immediate decline in the capital ratio.

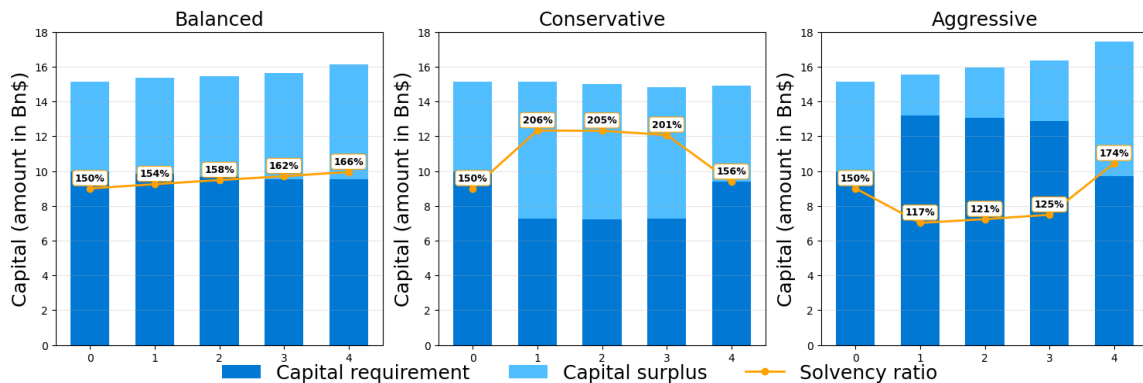
Conversely, the conservative strategy reduces market risk, leading to an immediate easing of capital requirements and a corresponding increase in the capital ratio.

During the four-year period, higher or lower investment earnings are added to the company’s capital in the opposite direction to the change in capital required. As shown in Figure 8, the effect on the capital required outweighs the effect on investment earnings.

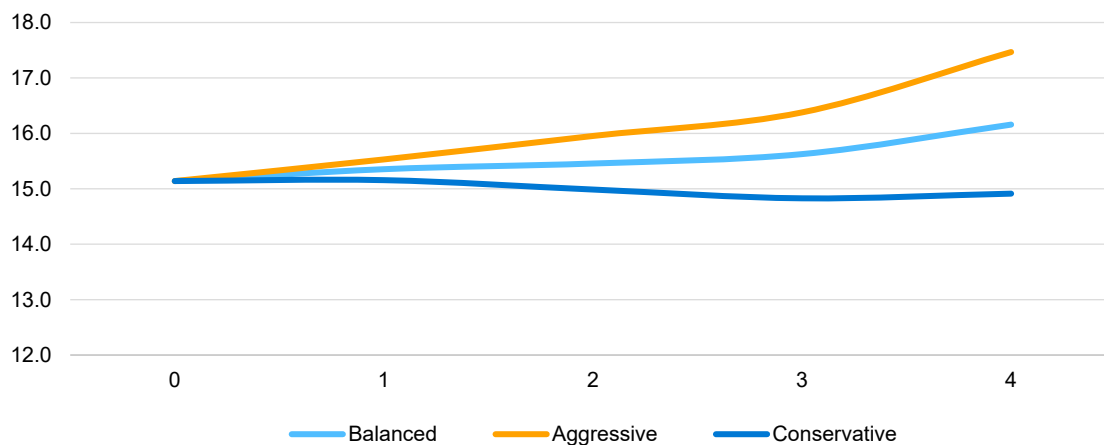
At the end of the four-year projection, once the portfolio reverts to the balanced allocation, capital requirements return to the same level, facilitating a comparison of the respective strategies.

We also examine the evolution of the key components underlying the capital ratio under each investment strategy.

FIGURE 9: PROJECTION OF THE CAPITAL RATIO, SURPLUS CAPITAL, AND REQUIRED CAPITAL BY ASSET STRATEGY



From Figure 9, the total capital of the company (required capital plus surplus capital) moves in line with different asset strategies. The higher expected economic return results in higher total capital.

FIGURE 10: COMPARISON OF PROJECTED TOTAL CAPITAL (REQUIRED + SURPLUS IN BN\$) BY ASSET STRATEGY

The simple deterministic analysis of the investment options suggests that over a four-year horizon, the aggressive strategy appears preferable:

- It increases own funds by the end of the projection
- It improves the capital ratio once the portfolio returns to the balanced allocation
- With all of the above, it remains above the regulatory threshold of 100% throughout the entire period

However, this conclusion is based on a single deterministic path. The deterministic projection does not reflect the volatility and risks associated with achieving this outcome, most notably the possibility of the capital ratio falling below 100% or the company failing to meet other targets.

Stochastic projections

A stochastic projection of economic returns provides a full distribution of possible outcomes and allows quantification of the risks.

At this stage, a new dimension is added to the analysis when investment returns are assumed to be stochastic rather than deterministic. The assumed investment return is, in practice, a volatile variable shaped by a wide range of dynamic and interrelated factors.

By simulating a wide range of economic scenarios, it becomes possible to assess not only the expected value of the solvency ratio but also the distribution of outcomes and their associated probabilities, thereby enabling a quantitative evaluation of the underlying risks.

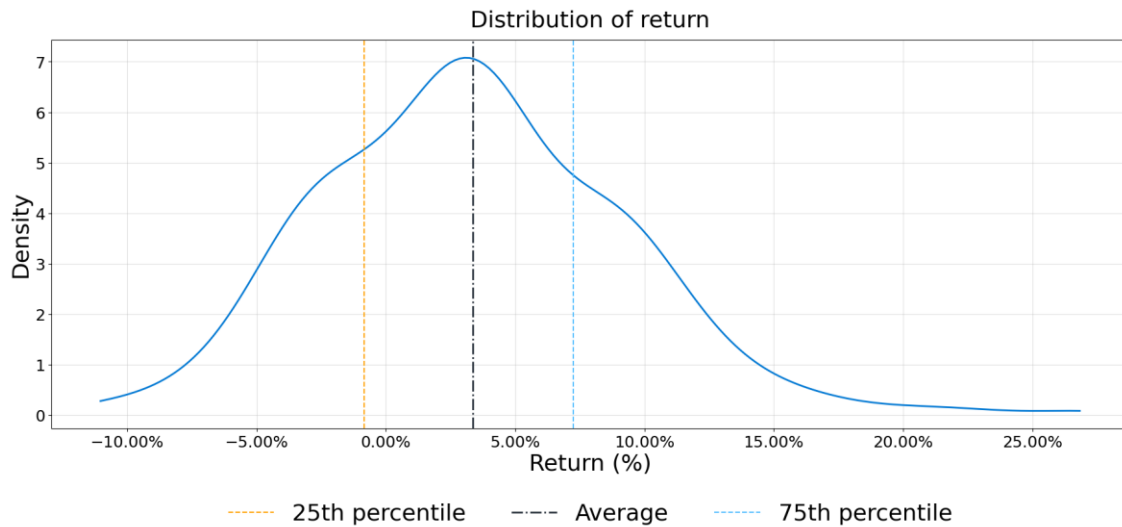
Market scenarios: Economic Scenario Generator

To run a stochastic forecast, economic scenarios are required; these can be built with an Economic Scenario Generator (ESG).

The economic scenarios reflect the expected real-world investment returns and volatility of the various asset classes. In addition, the scenarios include the correlation effects between asset classes. In this case, 5,000 possible investment return scenarios were generated, each representing a potential projection of market conditions over the coming years.

The average across all scenarios converges to the deterministic returns assumed in the base projection (Figure 7), showing consistency with the original assumptions while revealing the distribution around that average.

FIGURE 11: DISTRIBUTION OF INVESTMENT RETURNS FOR THE NOSTRO PORTFOLIO (NON-PARTICIPATING)

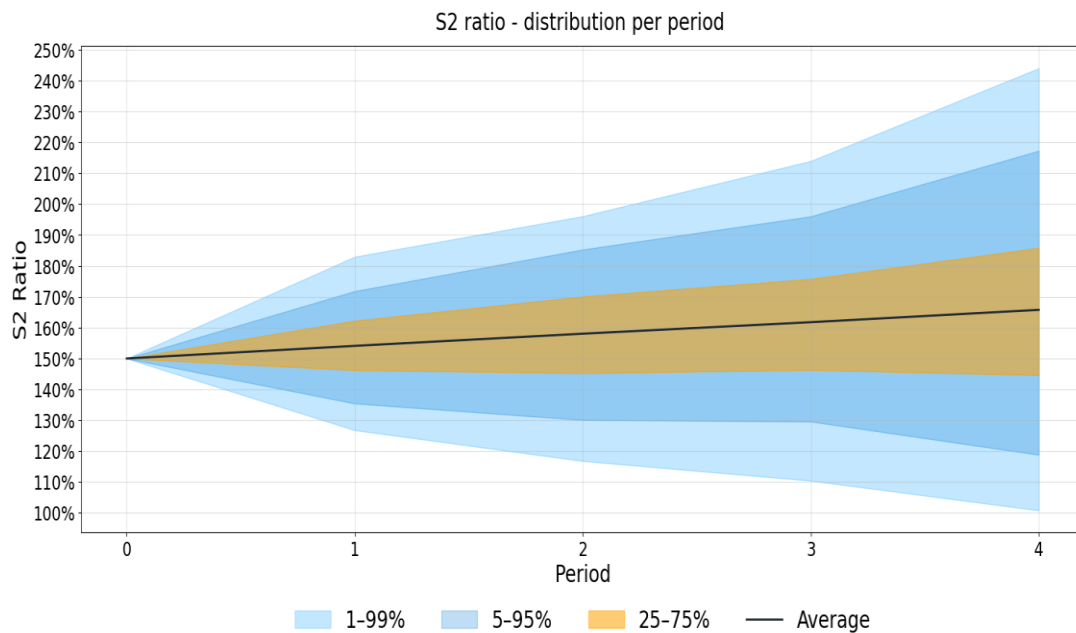


The stochastic projection

The projection model is rerun with the set stochastic scenarios over the projection period, producing a corresponding set of outcomes for the company's solvency position.

Unlike the single result of the deterministic projection, the stochastic projection produces a range of possible results, which can be displayed in a graph to show the distribution and key percentiles. This highlights volatility in the projected results as shown in Figure 12.

FIGURE 12: DISTRIBUTION OF THE SOLVENCY RATIO, BASED ON 5,000 STOCHASTIC SIMULATIONS



The stochastic approach allows us to identify not only the average solvency ratio but also the level of volatility and uncertainty embedded in the projection.

This perspective enables quantification of the probability of falling below desired solvency levels, assessing the potential for improvement in the capital ratio, and evaluating the company's overall financial resilience.

The distribution of results can also be displayed using a boxplot, as demonstrated in Figure 13. The boxplot shows the average value, the central range (i.e., 50% of the outcomes from the 25th to 75th percentiles), and the outer bounds of the 1% and 99% extreme results.

FIGURE 13: CAPITAL RATIO DISTRIBUTION (BOXPLOT)

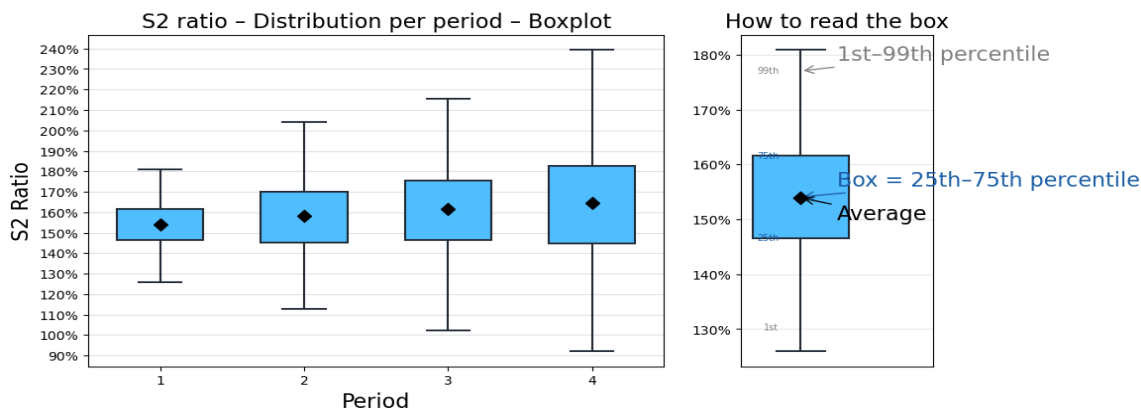


Figure 13 provides a good comparable measure of the volatility and the extent of extreme outcomes in both tails.

RISK APPETITE

The distribution of stochastic results can be used to derive statistical risk measures such as Value at Risk (VaR), which identifies the shortfall threshold, and these can be analyzed for various levels. For example, the 1%, 5%, and 10% percentiles represent lower-tail outcomes that the capital ratio falls below in only 1%, 5%, or 10% of scenarios, respectively. Equivalently, these percentiles represent exceedance probabilities of 99%, 95%, and 90%.

These metrics allow us to quantify tail risk, measure the company's shortfall exposure, and assess the adequacy of the financial buffer relative to the firm's stated risk appetite.

A practical application is evaluating the probability of meeting key solvency targets, as in the following examples:

- Maintaining a capital ratio above the regulatory minimum (i.e. 100%)
- Remaining above the company's dividend-distribution threshold (e.g., 130%)

Using the stochastic simulations, we can calculate the probability of shortfall with respect to each target, thereby providing a quantitative assessment of the risk and assessing whether the company's risk appetite is satisfied.

For example, if the company has a risk tolerance stating that the solvency ratio may fall below 100% in 1% of cases and that the solvency ratio may fall below 130% in 10% of cases, the company's risk appetite has been met, as shown in Figure 14.

FIGURE 14: PROBABILITY OF CAPITAL RATIO SHORTFALL FOR SELECTED THRESHOLDS - BALANCED STRATEGY

	YEAR 1	YEAR 2	YEAR 3	YEAR 4
Risk tolerance for ratio <100% = 1%	✓ 0.0%	✓ 0.0%	✓ 0.0%	✓ 0.7%
Risk tolerance for ratio <130% = 10%	✓ 1.5%	✓ 4.9%	✓ 5.6%	✓ 9.9%

Using the same approach, we can assess additional targets, as in the following examples:

- **Annual profit stability**—for example, a 70% probability that annual profit remains within a defined target range
- **Achieving a capital ratio of 170%**—for example, a 50% probability that the company reaches a 170% capital ratio

The stochastic perspective links the capital projection to the company’s risk appetite. It offers a structured framework for evaluating whether the volatility embedded in the results aligns with the firm’s stated risk appetite.

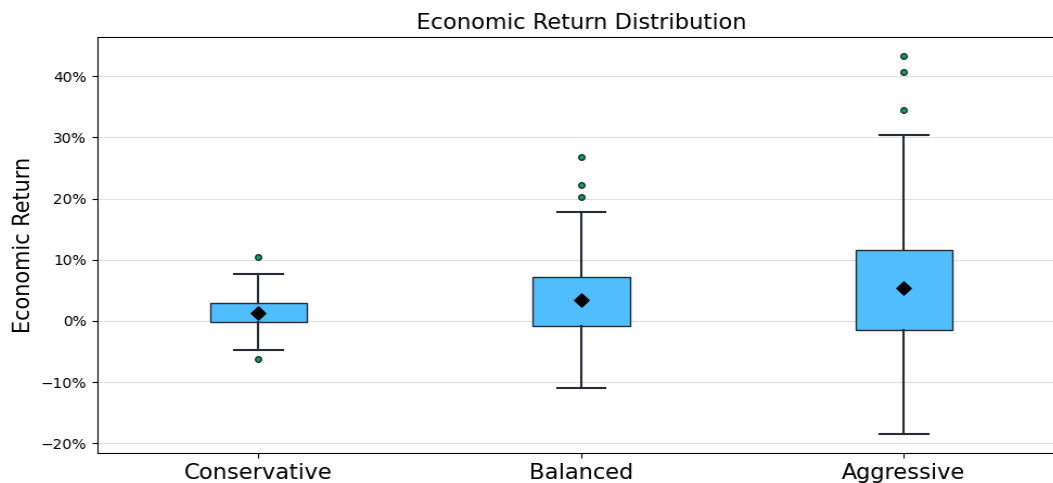
This approach does not replace broader business or managerial considerations; it adds a quantitative foundation—a mathematical framework for measuring and interpreting risk.

Assessment of alternative asset allocations - Revisited

It is possible to revisit the alternative asset allocations and assess the volatility, which was missing from the deterministic projections of the various options.

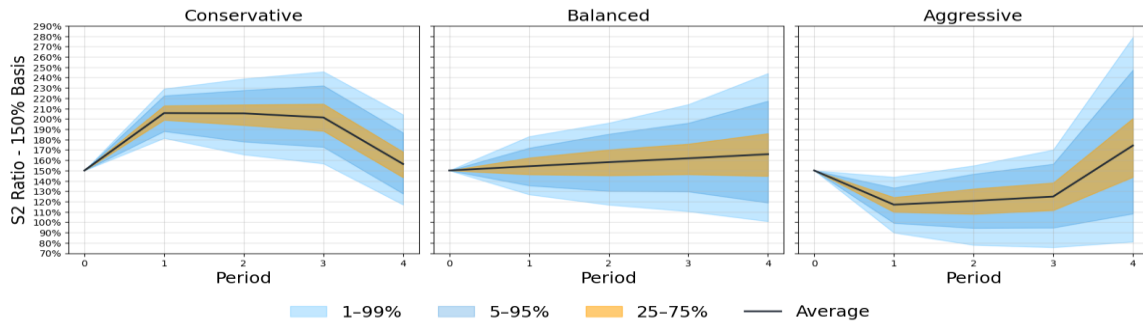
The riskier asset strategy increases the range of investment return scenarios, as shown in Figure 15.

FIGURE 15: RETURN DISTRIBUTIONS BY INVESTMENT STRATEGY



Alternative investment scenarios are used to reproduce the projected solvency results, with the spread of outcomes over the four-year projection horizon being shown in Figure 16.

FIGURE 16: DISTRIBUTION OF THE SOLVENCY RATIO BY INVESTMENT STRATEGY, BASED ON 5,000 STOCHASTIC SIMULATIONS⁴

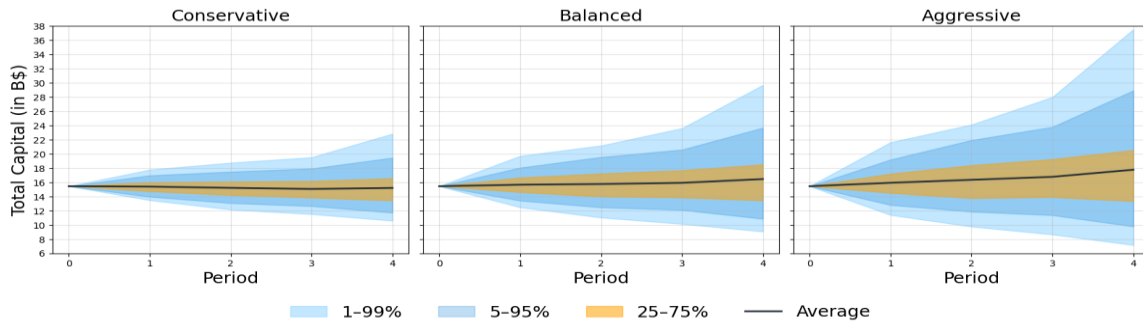


It should be noted that at the end of year four, the investment portfolio is assumed to revert to the original balanced allocation, removing the distortion from capital requirements, which, in turn, adjusts the solvency ratio accordingly.

The stochastic analysis highlights the risks embedded in the two investment strategies. The aggressive strategy, although appearing preferable under a deterministic viewpoint, has higher volatility, especially at the end of the fourth year.

An alternative approach is to compare the distribution of Total Capital for the investment options. The volatility of results increases as the investment strategy becomes riskier. The stochastic approach allows the company to quantify the risk and assess whether the risk level is acceptable.

FIGURE 17: DISTRIBUTION OF TOTAL CAPITAL (IN BN\$) BY INVESTMENT STRATEGY - 5,000 STOCHASTIC SIMULATIONS



4. It is worth noting that we expected to observe an increase in the standard deviation of the capital ratio in the first three years of the projection when moving from the balanced strategy to the aggressive strategy, similar to the increase observed when moving from the conservative to the balanced strategy. In the above projections, there is no observable increase in the volatility of the solvency ratio in years one to three. This is because the metric being examined is a ratio. More specifically, higher volatility of own funds is divided by a higher capital requirements level causing the ratio volatility to be dampened, even though the underlying own funds are more volatile.

RISK APPETITE

From the stochastic runs, it is possible to assess whether the alternative asset strategies fall within the company's risk appetite. For the same risk tolerance levels as described above, Figure 18 is expanded to include the conservative and aggressive strategies.

FIGURE 18: RISK-APPETITE MAPPING TO ASSET-ALLOCATION STRATEGIES

STRATEGY		LOW RISK APPETITE			
		YEAR 1	YEAR 2	YEAR 3	YEAR 4 - RETURN TO BALANCED
Conservative	Risk tolerance for ratio <100% = 1%	✓ 0.0%	✓ 0.0%	✓ 0.0%	✓ 0.0%
Balanced		✓ 0.0%	✓ 0.0%	✓ 0.0%	✓ 0.7%
Aggressive		✗ 8.0%	✗ 9.8%	✗ 11.1%	✗ 3.0%
Conservative	Risk tolerance for ratio <130% = 10%	✓ 0.0%	✓ 0.0%	✓ 0.0%	✓ 7.2%
Balanced		✓ 1.5%	✓ 4.9%	✓ 5.6%	✓ 9.9%
Aggressive		✗ 90.1%	✗ 71.2%	✗ 61.1%	✗ 14.3%

Figure 18 translates the investment strategy to the company's risk profile. It shows that the aggressive strategy does not meet the company's risk appetite levels.

A company with a higher risk appetite can adopt the aggressive investment strategy. Alternatively, a company may set its targets on a one- or two-year time horizon, making different conclusions. The company may instead devise a less aggressive strategy and re-evaluate the outcomes.

In summary, incorporating a stochastic model allows evaluation of possible capital outcomes under a given strategy and an alignment of the investment strategy with the company's risk appetite.

One size fits all?

The optimal strategy is determined not solely by the firm's risk appetite, but also by its current capital position, market conditions, and broader business environment.

The results and conclusions depend on a range of factors, including:

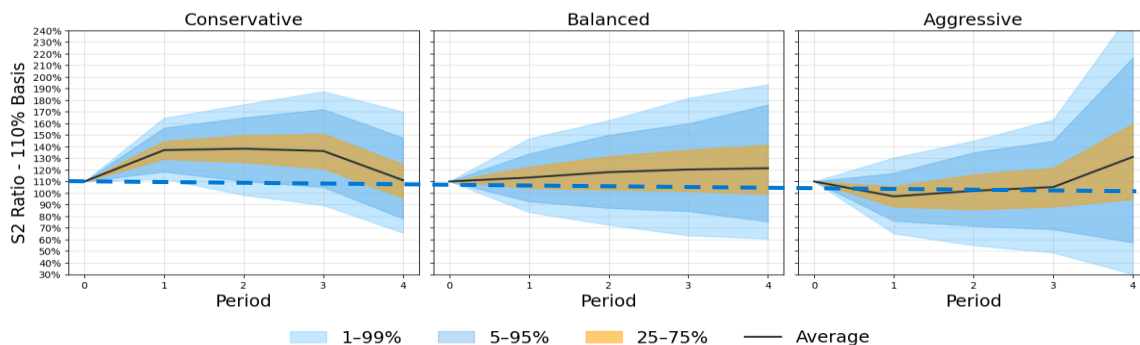
- Economic environment
- Dividend-distribution expectations
- Composition of the liability portfolio (investment risk exposure and duration)
- Liquidity requirements
- Regulatory requirements
- Current solvency ratio

These differences shape how risks and exposures are perceived and ultimately affect the conclusions drawn from the analysis.

For example, if the current capital ratio is 110%, the conclusions will be different.

Figure 19 shows the results of the stochastic runs for a company more exposed to the risk of the solvency ratio falling below 100%.

FIGURE 19: SOLVENCY RATIO DISTRIBUTIONS BY INVESTMENT STRATEGIES - INITIAL CAPITAL RATIO OF 110%



The 100% solvency ratio (dashed line) is breached with increasing probability as the investment strategy becomes riskier.

In the risk-appetite matrix, no asset strategy fully meets the company’s risk appetite over the four-year horizon.

FIGURE 20: RISK-APPETITE MAPPING ACROSS INVESTMENT STRATEGIES FOR A 110% CAPITAL RATIO

STRATEGY		YEAR 1	YEAR 2	YEAR 3	YEAR 4 - BACK TO BALANCED
Conservative	Risk tolerance for ratio <100% = 1%	✓ 0.0%	✓ 1.6%	✓ 2.6%	✗ 33.4%
Balanced		✗ 17.7%	✗ 19.6%	✗ 22.2%	✗ 25.8%
Aggressive		✗ 58.0%	✗ 45.6%	✗ 41.2%	✗ 28.9%

The conservative strategy is the closest match and the most suitable of the three options.

Furthermore, at the end of year four, for comparison purposes, the investment strategy was changed back to the balanced strategy, but that caused the year four solvency ratio to fall. To meet the solvency targets, it would be better for the company to remain with the conservative asset allocation and not switch back to the original balanced strategy.

Conclusion

Extending the deterministic framework to a stochastic framework adds another dimension to the analysis of solvency projections. Rather than relying on a single prescribed scenario, the stochastic approach produces a full distribution of potential outcomes, providing a more comprehensive understanding of the uncertainty and volatility for a company. This is in addition to some improvement in accuracy of the mean outcome by incorporating non-linear effects that invariably exist.

Evaluating a wide range of solvency paths enables deeper insight into the risks embedded in investment strategies and their long-term implications for the company’s financial resilience.

Further Analysis

Although the stochastic framework introduces additional complexity and necessitates more advanced modeling techniques, it also lays the groundwork for extending the analysis to several important areas, including:

- Risk-mitigation strategies, such as dynamic hedging and protection against adverse market movements
- Dynamic asset-allocation strategies that respond to changes in capital levels, market conditions, or portfolio composition
- Assessing IFRS 17 profitability under stochastic scenarios
- Assessing the appropriateness of the Solvency II standard-formula market-risk capital requirements relative to the actual level of risk observed in the local market

Ultimately, the richer viewpoint offered by stochastic modeling enables a natural progression from risk assessment to optimization. It becomes possible to identify asset-allocation strategies that maximize economic value while remaining aligned with the company's defined risk appetite. This, in turn, supports the construction of an Efficient Frontier, providing a robust framework for determining the optimal asset mix for each risk level and guiding strategic capital-management decisions.

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